

# QCD factorization beyond leading twist in exclusive processes: $\rho_T$ -meson production

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Exclusive processes in hard electroproduction are one of the best place for understanding the factorization properties of QCD. The HERA experiment recently provided precise data for  $\rho$  electroproduction, including all spin density matrix elements. From QCD, it is expected that such a process should factorize into a hard (calculable) coefficient function, and hadronic ( $p$  and  $\rho$ ) matrix elements. Such a factorization is up to now only proven for a longitudinally polarized  $\rho$ . Within the  $k_T$ -factorization approach (valid at large  $s_{\gamma^*p}$ ), we evaluate the impact factor of the transition  $\gamma^* \rightarrow \rho_T$  taking into account the twist 3 contributions. We show that a gauge invariant expression is obtained with the help of QCD equations of motion. More generally, relying on these equations and on the invariance under rotation on the light-cone of the factorized amplitude, the non-perturbative Distribution Amplitudes can be reduced to a minimal set. This opens the way to a consistent treatment of factorization for exclusive processes with a transversally polarized vector meson. We prove the equivalence of two proposed parametrizations of the  $\rho_T$  distribution amplitudes.

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## 1. Introduction

Studies of hard exclusive reactions rely on the factorization properties of the leading twist amplitudes [1]. The leading twist distribution amplitude (DA) of a transversally polarized vector meson is chiral-odd, and hence decouples from hard amplitudes even when another chiral-odd quantity is involved [2] unless in reactions with more than two final hadrons [3]. Thus transversally polarized  $\rho$ -meson production is generically governed by twist 3 contributions for which a pure collinear factorization fails due to the appearance of end-point singularities [4,5]. The meson quark gluon structure within collinear factorization may be described by Distribution Amplitudes (DAs), classified in [6]. Measurements [7] of the  $\rho_T$ -meson production amplitude in photo and electro-production show that it is by no means negligible. We consider here the case of very high energy collisions at colliders, for which future progress may come from real or virtual photon photon collisions [8, 9]. In the literature there are two approaches to the factorization of the scattering amplitudes in exclusive processes at leading and higher twists. The first approach [5, 10], the Light-Cone Collinear Factorization (LCCF), extends the inclusive approach [11] to exclusive processes, dealing with the factorization in the momentum space around the dominant light-cone direction. On the other hand, there exists a Covariant Collinear Factorization (CCF) approach in coordinate space successfully applied in [6] for a systematic description of DAs of hadrons carrying different twists. We show [12] that these two descriptions are equivalent at twist 3, and illustrate this by calculating within both methods the impact factor  $\gamma^* \rightarrow \rho_T$ , up to twist 3 accuracy.

## 2. LCCF factorization of exclusive processes

The amplitude for the exclusive process  $A \rightarrow \rho B$  is, in the momentum representation and in axial gauge reads ( $H$  and  $H_\mu$  are 2- and 3-parton coefficient functions, respectively)

$$\mathcal{A} = \int d^4\ell \operatorname{tr} \left[ H(\ell) \Phi(\ell) \right] + \int d^4\ell_1 d^4\ell_2 \operatorname{tr} \left[ H_\mu(\ell_1, \ell_2) \Phi^\mu(\ell_1, \ell_2) \right] + \dots \quad (2.1)$$

In (2.1), the soft parts  $\Phi$  are the Fourier-transformed 2- or 3-parton correlators which are matrix elements of non-local operators. To factorize the amplitude, we choose the dominant direction around which we decompose our relevant momenta and we Taylor expand the hard part. Let  $p \sim p_\rho$  and  $n$  be two light-cone vectors ( $p \cdot n = 1$ ). Any vector  $\ell$  is then expanded as

$$\ell_{i\mu} = y_i p_\mu + (\ell_i \cdot p) n_\mu + \ell_{i\mu}^\perp, \quad y_i = \ell_i \cdot n, \quad (2.2)$$

and the integration measure in (2.1) is replaced as  $d^4\ell_i \rightarrow d^4\ell_i dy_i \delta(y_i - \ell \cdot n)$ . The hard part  $H(\ell)$  is then expanded around the dominant  $p$  direction:

$$H(\ell) = H(y p) + \left. \frac{\partial H(\ell)}{\partial \ell_\alpha} \right|_{\ell=y p} (\ell - y p)_\alpha + \dots \quad (2.3)$$

where  $(\ell - y p)_\alpha \approx \ell_\alpha^\perp$  up to twist 3. To obtain a factorized amplitude, one performs an integration by parts to replace  $\ell_\alpha^\perp$  by  $\partial_\alpha^\perp$  acting on the soft correlator. This leads to new operators containing

transverse derivatives, such as  $\bar{\psi} \partial^\perp \psi$ , thus requiring additional DAs  $\Phi^\perp(l)$ . Factorization is then achieved by Fierz decomposition on a set of relevant Dirac  $\Gamma$  matrices, and we end up with

$$\mathcal{A} = \text{tr}[H_{q\bar{q}}(y)\Gamma] \otimes \Phi_{q\bar{q}}^\Gamma(y) + \text{tr}\left[H_{q\bar{q}}^{\perp\mu}(y)\Gamma\right] \otimes \Phi_{q\bar{q}\mu}^{\perp\Gamma}(y) + \text{tr}\left[H_{q\bar{q}g}^\mu(y_1, y_2)\Gamma\right] \otimes \Phi_{q\bar{q}g\mu}^\Gamma(y_1, y_2), \quad (2.4)$$

where  $\otimes$  is the  $y$ -integration. Although the fields coordinates  $z_i$  are on the light-cone in both LCCF and CCF parametrizations of the soft non-local correlators,  $z_i$  is along  $n$  in LCCF while arbitrary in CCF. The transverse physical polarization of the  $\rho$ -meson is defined by the conditions

$$e_T \cdot n = e_T \cdot p = 0. \quad (2.5)$$

Keeping all the terms up to the twist-3 order with the axial (light-like) gauge,  $n \cdot A = 0$ , the matrix elements of quark-antiquark nonlocal operators for vector and axial-vector correlators without and with transverse derivatives, with  $\overleftrightarrow{\partial}_\rho = \frac{1}{2}(\overrightarrow{\partial}_\rho - \overleftarrow{\partial}_\rho)$ , can be written as (here,  $z = \lambda n$ )

$$\begin{aligned} \langle \rho(p_\rho) | \bar{\psi}(z) \gamma_5 \gamma_\mu \psi(0) | 0 \rangle &= m_\rho f_\rho i \int_0^1 dy \exp[iy p \cdot z] \phi_A(y) \varepsilon_{\mu\alpha\beta\delta} e_T^{*\alpha} p^\beta n^\delta \\ \langle \rho(p_\rho) | \bar{\psi}(z) \gamma_5 \gamma_\mu i \overleftrightarrow{\partial}_\alpha^T \psi(0) | 0 \rangle &= m_\rho f_\rho i \int_0^1 dy \exp[iy p \cdot z] \phi_A^T(y) p_\mu \varepsilon_{\alpha\lambda\beta\delta} e_T^{*\lambda} p^\beta n^\delta, \end{aligned} \quad (2.6)$$

for the axial case, where  $y$  ( $\bar{y}$ ) is the quark (antiquark) momentum fraction. Two analogous correlators are needed to describe gluonic degrees of freedom, introducing  $B$  and  $D$  DAs. One thus needs 7 DAs:  $\phi_1$  (twist-2),  $B$  and  $D$  (genuine (dynamical) twist-3) and  $\phi_3$ ,  $\phi_A$ ,  $\phi_1^T$ ,  $\phi_A^T$  (contain both parts: kinematical (à la Wandzura-Wilczek) twist-3 and genuine (dynamical) twist-3). These DAs are related by 2 Equations of Motions (EOMs) and 2 equations arising from the invariance of  $\mathcal{A}$  under rotation on the light-cone. Indeed, this invariance with respect to  $n$  does not involve the hard part of  $\mathcal{A}$ , and therefore implies constraints on the soft part, i.e. on the DAs. We thus have only 3 independent DAs  $\phi_1$ ,  $B$  and  $D$ , which fully encode the non-perturbative content of the  $\rho$  at twist 3.

The original CCF parametrizations of the  $\rho$  DAs [6] also involve 3 independent DAs, defined through 4 correlators related by EOMs. For example, the 2-parton axial-vector correlators reads,

$$\langle \rho(p_\rho) | \bar{\psi}(z) [z, 0] \gamma_\mu \gamma_5 \psi(0) | 0 \rangle = \frac{1}{4} f_\rho m_\rho \varepsilon_\mu^{\alpha\beta\gamma} e_{T\alpha}^* p_\beta z_\gamma \int_0^1 dy e^{iy(p \cdot z)} g_\perp^{(a)}(y), \quad (2.7)$$

$[z_1, z_2] = P \exp \left[ ig \int_0^1 dt (z_1 - z_2)_\mu A^\mu(t z_1 + (1-t) z_2) \right]$  being the Wilson line. Denoting the meson polarization vector by  $e$ ,  $e_T$  is here defined to be orthogonal to the light-cone vectors  $p$  and  $z$ :

$$e_{T\mu} = e_\mu - p_\mu \frac{e \cdot z}{p \cdot z} - z_\mu \frac{e \cdot p}{p \cdot z}, \quad (2.8)$$

Thus  $e_T$  (2.8) in CCF and  $e_T$  (2.5) in LCCF differ since  $z$  does not generally point in the  $n$  direction.

### 3. $\gamma^* \rightarrow \rho_T$ Impact factor up to twist three accuracy in LCCF and CCF

We have calculated, in both LCCF and CCF, the forward impact factor  $\Phi^{\gamma^* \rightarrow \rho}$  of the subprocess  $g + \gamma^* \rightarrow g + \rho_T$ , defined as the integral of the discontinuity in the  $s$  channel of the off-shell

S-matrix element  $\mathcal{S}_\mu^{\gamma_T^* g \rightarrow \rho_T g}$ . In LCCF, one computes the diagrams perturbatively in a fairly direct way, which makes the use of the CCF parametrization [6] less practical. We need to express the impact factor in terms of hard coefficient functions and soft parts parametrized by the light-cone matrix elements. The standard technique here is an operator product expansion on the light cone, which gives the leading term in the power counting. Since there is no operator definition for an impact factor, we have to rely on perturbation theory. The primary complication here is that the  $z^2 \rightarrow 0$  limit of any single diagram is given in terms of light-cone matrix elements without any Wilson line insertion between the quark and gluon operators ("perturbative correlators"), like  $\langle \rho(p_\rho) | \bar{\psi}(z) \gamma_\mu \psi(0) | 0 \rangle$ . Despite working in the axial gauge one cannot neglect effects coming from the Wilson lines since the two light cone vectors  $z$  and  $n$  are not identical and thus, generically, Wilson lines are not equal to unity. Nevertheless in the axial gauge the contribution of each additional parton costs one extra power of  $1/Q$ , allowing the calculation to be organized in a simple iterative manner expanding the Wilson line. At twist 3, we need to keep the contribution  $[z, 0] = 1 + ig \int_0^1 dt z^\alpha A_\alpha(zt)$  and to care about the difference between the physical  $\rho_T$ -polarization (2.5) from the formal one (2.8). At twist 3-level the net effect of the Wilson line when computing our impact factor is just a renormalization of the DA  $g_\perp^a$  of (2.7), and similarly for the vector case.

Based on the solution of the EOMs and  $n$ -independence set of equations, our two LCCF and CCF results are identical; they are gauge invariant due to a consistent inclusion of fermionic and gluonic degrees of freedom and are free of end-point singularities, due to the  $k_T$  regulator.

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